

# Statistics

## Lecture 12



Feb 19-8:47 AM

(SG 19)

Consider the Population 2, 4, 6, and 8.

Clear all lists  $\boxed{2nd} \boxed{+} \boxed{4:clearAllLists} \boxed{Enter}$

store 2, 4, 6, 8 in L1  $\boxed{STAT} \boxed{Edit} \boxed{1:Edit}$

$\boxed{STAT} \boxed{\rightarrow} \boxed{CALC} \boxed{1-VarStats}$

use  $\boxed{1-VarStats}$  with L1 only to find

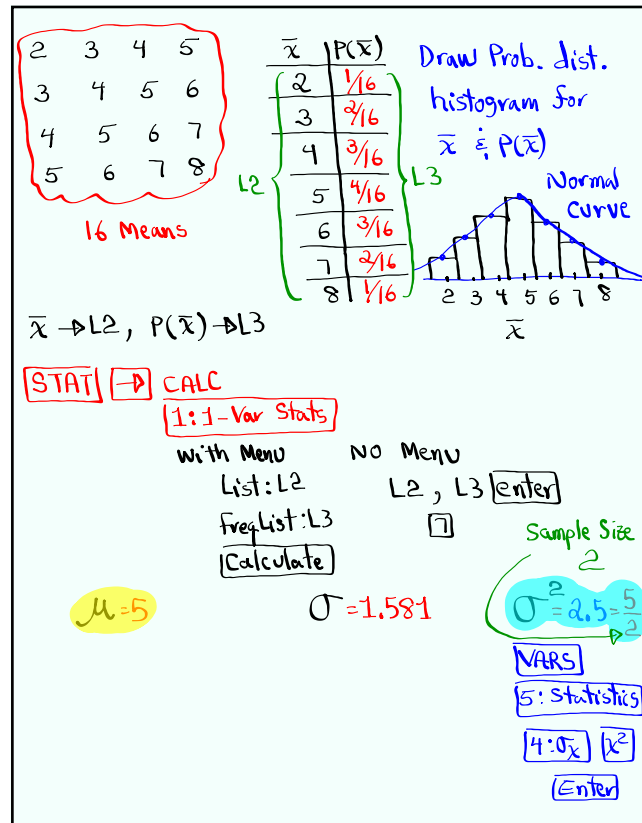
$\mu = \bar{x} = 5$        $\sigma = \sigma_x = 2.236$        $\sigma^2 = \sigma_x^2 = 5$

Now take all Samples of Size 2 with replacement from this data.

$\boxed{VARs} \boxed{5:Statistics} \boxed{4:\sigma_x} \boxed{X^2} \boxed{Enter}$

2,2	2,4	2,6	2,8	
4,2	4,4	4,6	4,8	Now find $\bar{x}$ of each Sample.
6,2	6,4	6,6	6,8	2   3   4   5
8,2	8,4	8,6	8,8	3   4   5   6
				4   5   6   7
				5   6   7   8

Jul 17-4:33 PM



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Consider the Population 2, 4, 6, 8, 10.

Clear all lists

Store 2, 4, 6, 8, 10 in L1

Use  $\rightarrow$  1-Var Stats with L1 only to find

$\mu = 6$   $\sigma = 2.828$   $\sigma^2 = 8$

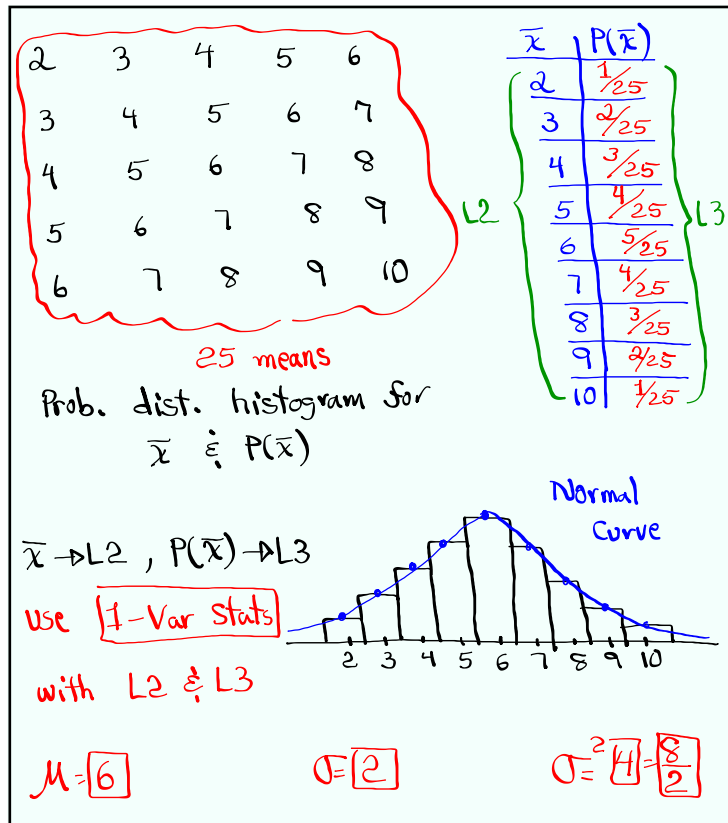
let's take all samples of Size 2 from this data with replacement

2,2	2,4	2,6	2,8	2,10
4,2	4,4	4,6	4,8	4,10
6,2	6,4	6,6	6,8	6,10
8,2	8,4	8,6	8,8	8,10
10,2	10,4	10,6	10,8	10,10

now find  $\bar{x}$  of each sample

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

Jul 17-4:55 PM



Jul 17-5:03 PM

$\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

## Central-Limit Theorem

Given  $N(35, 6)$  Normal dist with  
 $\mu = 35, \sigma = 6$

If we take all Samples of Size 4.

$\mu_{\bar{x}} = \mu = \boxed{35}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{4}} = \frac{6}{2} = \boxed{3}$

## CLT

Jul 17-5:12 PM

Salaries of all nurses are normally dist. with  $\mu = \$6800$  &  $\sigma = \$500$

If we randomly selected groups of 16 nurses,

$$\mu_{\bar{x}} = \mu = \boxed{6800}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{500}{\sqrt{16}} = \frac{500}{4} \boxed{125}$$

CLT

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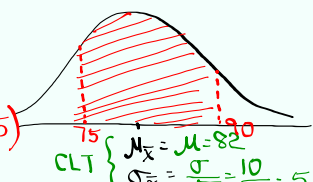
Scores of math exams are normally dist. with  $\mu = 82$  &  $\sigma = 10$ .  $N(82, 10)$

If we randomly select  $n=4$  exams,

find the Prob. that their mean  $\bar{x}$  is between 75 and 90.

$$P(75 < \bar{x} < 90) = \text{normalcdf}(75, 90, 82, 5) = \boxed{.864}$$

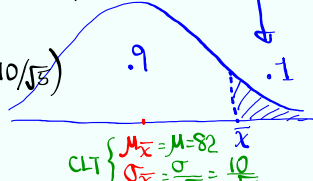
CLT  $\begin{cases} \mu_{\bar{x}} = \mu = 82 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{4}} = 5 \end{cases}$



for randomly selected groups of 5 exams, find the mean  $\bar{x}$  that separates the top 10% from the rest.

$$\bar{x} = \text{invNorm}(.9, 82, 10/\sqrt{5}) = 87.734 \approx \boxed{88}$$

CLT  $\begin{cases} \mu_{\bar{x}} = \mu = 82 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{5}} \end{cases}$



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Ages of students are normally dist. with mean of 32 yrs and standard dev. of 7 yrs.  $N(32, 7)$

If we randomly select  $n=3$  students, find the prob. that their  $\bar{x}$  mean age is more than 28 yrs.

$P(\bar{x} > 28)$

$= \text{normalcdf}(28, E99, 32, 7/\sqrt{3})$

$\text{CLT} \begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7}{\sqrt{3}} \end{cases}$

$\text{2nd} \rightarrow \boxed{.839}$

For randomly selected groups of 5, find the  $\bar{x}$  mean that separates the bottom 20% from the rest.

$\text{CLT} \begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{7}{\sqrt{5}} \end{cases}$

$\bar{x} = \text{invNorm}(.2, 32, 7/\sqrt{5})$

$\approx 29.365 \approx \boxed{29}$

Jul 17-5:32 PM

Credit Scores are normally dist. with the mean 700 and standard dev. of 80.

If we randomly select  $n=4$  group of 4, find the Prob. that their  $\bar{x}$  mean credit score is below 675 or above 725.  $N(700, 80)$

$P(\bar{x} < 675 \text{ OR } \bar{x} > 725)$

$= 1 - P(675 < \bar{x} < 725)$

$= 1 - \text{normalcdf}(675, 725, 700, 40)$

$\text{CLT} \begin{cases} \mu_{\bar{x}} = \mu = 700 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{80}{\sqrt{4}} = 40 \end{cases}$

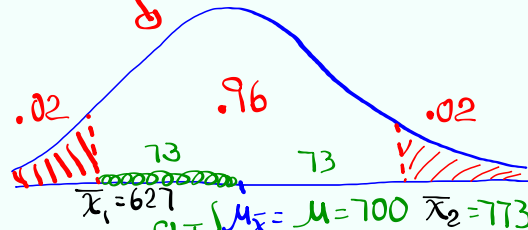
$= \boxed{.532}$

Jul 17-5:59 PM

Find  $\bar{x}_1$  &  $\bar{x}_2$ , rounded to whole numbers,  
for randomly selected groups of 5 that  
separate the middle 96% from the rest.

$$1 - .96 = .04$$

$$.04 \div 2 = .02$$



$$\bar{x}_1 = \text{invNorm}(.02, 700, 80/\sqrt{5})$$

$$\approx \boxed{627}$$

$$\bar{x}_2 = \text{invNorm}(.98, 700, 80/\sqrt{5}) \approx \boxed{773}$$

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Length of pregnancy is normally dist.  
with the mean of 250 days and standard  
dev. of 30 days.

$$N(250, 30)$$

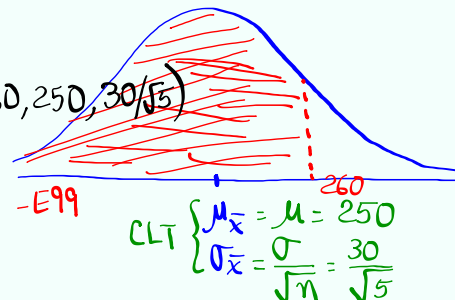
If we randomly select 5 pregnant women,  
find the prob. that their mean pregnancy time  
is below 260 days.

$$P(\bar{x} < 260)$$

$$= \text{normalcdf}(-E99, 260, 250, 30/\sqrt{5})$$

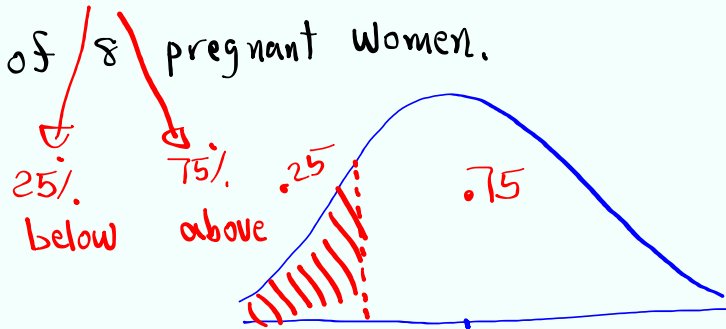
$$(-) \quad \boxed{2nd} \quad \boxed{V}$$

$$= \boxed{.772}$$



Jul 17-6:15 PM

find  $\bar{x} = Q_1$  for randomly selected groups of 8 pregnant women.



$$\bar{x} = \text{invNorm}(.25, 250, 30/\sqrt{8}) \quad \text{CLT} \quad \begin{cases} \mu_{\bar{x}} = \mu = 250 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{30}{\sqrt{8}} \end{cases}$$

$$\approx \boxed{243} \text{ days}$$

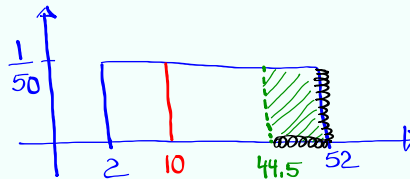
SG 20 ✓

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Consider a uniform Prob. dist. for all values from 2 to 52.  $52 - 2 = 50$

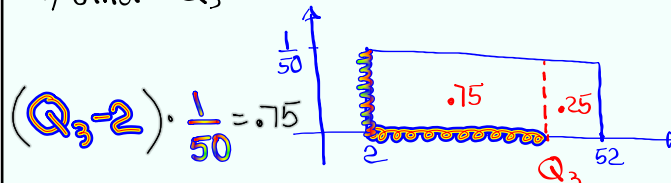
1) Draw  $\hat{\pi}$  label

2)  $P(\underline{x=10}) = 0$   
line



3)  $P(x > 44.5) = (52 - 44.5) \cdot \frac{1}{50} = \frac{7.5}{50} = \frac{15}{100} = \frac{3}{20}$

4) Find  $Q_3$  of the dist.



$$(Q_3 - 2) \cdot \frac{1}{50} = 0.75$$

$$Q_3 - 2 = 50(0.75)$$

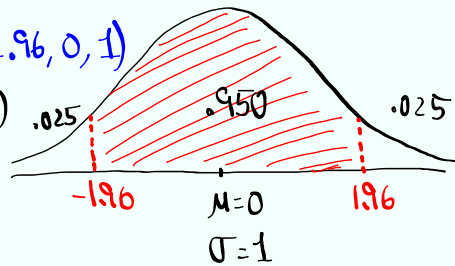
$$Q_3 = 2 + 50(0.75) = \boxed{39.5}$$

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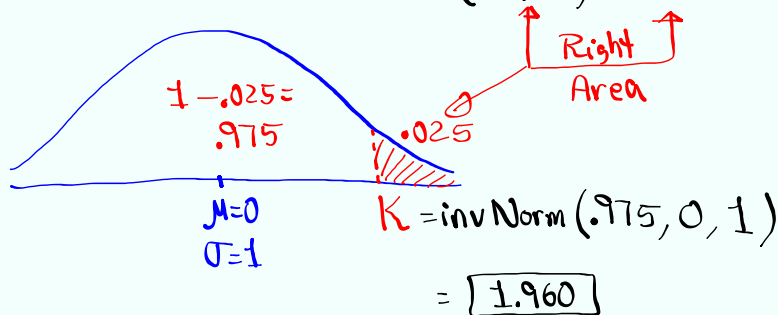
Find  $P(-1.96 < Z < 1.96)$

$= \text{normalcdf}(-1.96, 1.96, 0, 1)$

$= \boxed{.950}$   
95%



Find  $K$  such that  $P(Z > K) = .025$



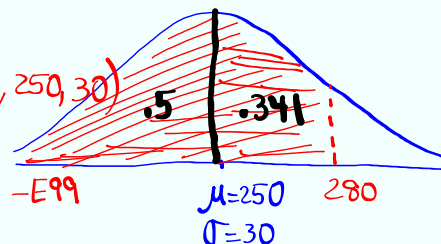
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Given  $N(250, 30)$

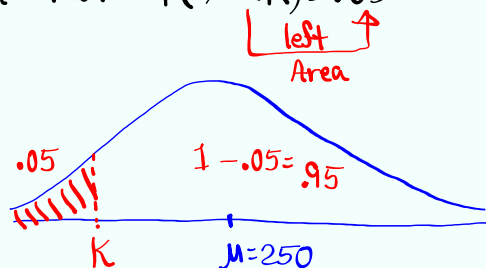
$P(X < 280)$

$= \text{normalcdf}(-E99, 280, 250, 30)$

$= \boxed{.841}$



Find  $K$  such that  $P(X < K) = .05$



$K = \text{invNorm}(.05, 250, 30) \approx 201$

Jul 17-6:45 PM

Consider a binomial Prob. dist with  $n=80$ ,  
and  $p=.75$ .

1)  $q = 1 - p = .25$     2)  $\mu = np = 60$

3)  $\sigma^2 = npq = 15$     4)  $\sigma = \sqrt{\sigma^2} = \sqrt{15} \approx 4$

5) Usual Range  $\mu \pm 2\sigma = 60 \pm 2(4) \Rightarrow 52 \text{ to } 68$

6)  $P(\overbrace{\# \text{Successes}}^x \geq 65) = P(X \geq 65)$   
 $= 1 - P(X \leq 64)$   
 ~~$= 1 - \text{binomcdf}(80, .75, 64)$~~   
 $= 1 - \text{binomcdf}(80, .75, 64)$   
 $= .121$

7)  $P(\# \text{Successes is between } 52 \text{ to } 68, \text{ inclusive})$

$P(52 \leq X \leq 68)$     Reduce by 1  
 $= \text{binomcdf}(80, .75, 68) - \text{binomcdf}(80, .75, 51)$   
 $\approx .973$

Jul 17-6:54 PM